of rhythm and gait were played by various factors, including rolling during walking, pitching in out-of-phase running, and the up-and-down motion of the trunk in in-phase running. The up-and-down motion was facilitated by the inclusion of spring elements.

We consider that this idea would work well also in the free locomotion of a quadruped robot because the leg phase switching inducing the self-excited motions of the trunk could be the essence of legged locomotion. We wish to emphasize that their understanding of the underlying mechanisms governing the emergence of legged locomotion has progressed nearly to a point where they can effectively incorporate these mechanisms into the controller of legged robots, while remaining faithful to the fundamental principles of legged locomotion.

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