4.4 Leg Controller

4.4.1 Overview

Each leg is actuated by one leg controller. Each LC has two phases, *swing* (sw) and *stance* (st), activated alternatively and the transfer of activity between them is controlled using conditions based on the load supported by the leg, as measured by the force sensor.

For the sake of convenience, each LC is associated with an oscillator[2]. The phase of the oscillator ϕ^i is used for the trajectory generation, as well as to express the phase relationship between the legs. The nominal value of the oscillator angular frequency ω is given by:

$$\hat{\omega} = 2\pi(1 - \hat{\beta})/\hat{T}_{sw}, \tag{4.1}$$

where β is the duty ratio and T_{sw} the swing phase duration (see Section 2.2.2).

4.4.2 Foot trajectory generation

In order to keep the presentation of the system as clear as possible, only the outline of the trajectory generation are presented in this chapter (detailed explanations are given in Appendix C). For each leg, the trajectories of the foot during both phases, represented in Figure 4.2, are expressed in the Cartesian coordinate system fixed to the trunk and centered at the hip joint. The subscripts $*_x$ and $*_y$, used in the following sections and in Appendix C, refer to the x and y coordinates in that coordinate system.

The position of the foot where the transition from swing to stance is desired to happen is called the *anterior extreme position* (AEP), while the position where it really happens is the *touchdown position* (TD), referred respectively as $\hat{\boldsymbol{r}}_{AEP}$ and $\boldsymbol{r}_{TD}$.

Similarly, the position where the transition from stance to swing is desired to happen is the *posterior extreme position* (PEP), while the position where it really happens is called the *liftoff position* (LO), referred respectively as $\hat{\boldsymbol{r}}_{PEP}$ and $\boldsymbol{r}_{LO}$.

The nominal phases at AEP and PEP are respectively:

$$\hat{\phi}_{AEP} = 2\pi(1 - \hat{\beta}) \quad , \quad \hat{\phi}_{PEP} = 0 \tag{4.2}$$

[2]Although an oscillator is associated to each LC, the CPG model constructed by these LCs is still sensor-dependent, as explained in Section 4.4.5.

4.4.2.1 Swing

At the beginning of the swing phase, ϕ is reset to $\hat{\phi}_{PEP}$ and the phase dynamics is subsequently given by:

$$\omega = \hat{\omega} + \omega_{mod} \qquad \text{and} \qquad \dot{\phi} = \begin{cases} \omega & \text{if } 0 \leq \phi < \hat{\phi}_{AEP} \\ 0 & \text{if } \phi \geq \hat{\phi}_{AEP} \end{cases} \tag{4.3}$$

where ω_{mod} can be used to modulate the swing phase duration (it is equal to zero except when it is set by the ascending coordination mechanism, see Section 5.3). The trajectory $\boldsymbol{r}_{sw}(\phi)$ satisfies:

$$\boldsymbol{r}_{sw}(0) = \boldsymbol{r}_{LO} \qquad \boldsymbol{r}_{sw}(\hat{\phi}_{AEP}) = \hat{\boldsymbol{r}}_{AEP} \tag{4.4}$$

with:

$$\hat{r}_{AEP,x} = \hat{L}_{str}/2 \tag{4.5}$$

$$\hat{r}_{AEP,y} = \hat{H} - \Delta y_{AEP} \tag{4.6}$$

where L_{str} is the stride length (see Section 2.2.2), H the body height and Δy_{AEP} an offset of the vertical coordinate of the AEP position (as explained in Section 4.5.4.1, this parameters influences the phase difference between ipsilateral legs of the emergent walking pattern).

4.4.2.2 Stance

At the beginning of the stance phase, ϕ is reset to $\hat{\phi}_{AEP}$ and the phase dynamics is subsequently given by:

$$\omega = \hat{\omega} \qquad \text{and} \qquad \dot{\phi} = \begin{cases} \omega & \text{if } \hat{\phi}_{AEP} \leq \phi < 2\pi \\ 0 & \text{if } \phi \geq 2\pi \end{cases} \tag{4.7}$$

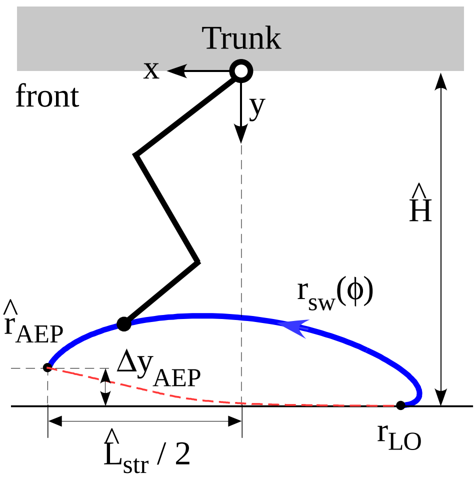
The trajectory $\boldsymbol{r}_{st}(\phi)$ satisfies:

$$\boldsymbol{r}_{st}(\hat{\phi}_{AEP}) = \boldsymbol{r}_{TD} \qquad \boldsymbol{r}_{st}(2\pi) = \hat{\boldsymbol{r}}_{PEP} \tag{4.8}$$

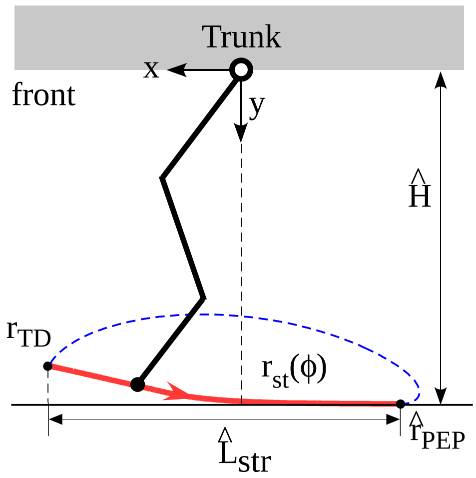
with:

$$\hat{r}_{PEP,x} = r_{TD,x} - \hat{L}_{str} \tag{4.9}$$

$$\hat{r}_{PEP,y} = \hat{H} \tag{4.10}$$



(a)



(b)

FIGURE 4.2: Stance and swing trajectories

The x component of the velocity $v_{st,x}$ is constant during the stance phase but for a short initial acceleration period, and is given by:

$$v_{st,x} = -\frac{\hat{L}_{str}}{\hat{T}_{st}} = -\frac{\hat{L}_{str}(1-\hat{\beta})}{\hat{\beta}\hat{T}_{sw}} \tag{4.11}$$

where T_{st} and T_{sw} are respectively the stance and swing phases durations and β the duty ratio.

On the other hand, the profile of the y component of the velocity is computed to recover from the height difference between $\boldsymbol{r}_{TD}$ and $\hat{\boldsymbol{r}}_{PEP}$. Different profiles have been tested and they lead to similar results as long as the velocity is large at the beginning of the stance phase and decreases fast enough afterward.

4.4.3 Sensory feedback and transition conditions

Transitions between the swing and the stance phases are regulated using leg loading information, where to the load supported by the leg is evaluated by the normal ground reaction force f_n measured by the foot force sensor. Additional conditions regarding the phase ϕ are added only to prevent a transition to occur at the very beginning of a phase that just started and avoid cascading transition triggering that would otherwise happen, as the foot is still touching the ground just after the termination of the stance phase and the leg not yet fully loaded just after the touchdown.

Conditions for the *transition from swing to stance* are:

- $f_n > 0$ (i.e. the foot touches the ground)

- $\phi > \hat{\phi}_{AEP}/2$

and for the *transition from stance to swing*:

- $f_n < \chi = \hat{\chi} + \chi_{mod}$
- $\phi > (\hat{\phi}_{AEP} + \pi/2)$

where χ_{mod} can be used to modulate the force threshold (as ω_{mod} in Equation 4.3, it is equal to zero, except when it is set by the ascending coordination mechanism, see Section 5.3). In the following experiments, $\hat{\chi}$ is set to 10 N which represents slightly less than one fourth of the simulation model weight.

4.4.4 Leg Motion Control

The trajectories of the joint angles (θ) are computed using the inverse kinematics model of the leg (see Appendix C.3). The torque at joint j is generated by local PD control as follows:

$$\Gamma_j = K_{Pj}(\hat{\theta}_j - \theta_j) + K_{Dj}(\dot{\hat{\theta}}_j - \dot{\theta}_j) \tag{4.12}$$

4.4.5 Implementation of the common principles

The leg controller implements the common principles defined in Chapter 2. The transitions between swing and stance phases are triggered using leg loading information, characterized by the normal ground reaction force.

As regards the CPG model issue, although an oscillator is associated to each LC, transitions are prevented as long as the conditions are not fulfilled by setting $\dot{\phi}$ to zero when ϕ reaches the maximum value allowed for the phase (i.e. $\hat{\phi}_{AEP}$ for the swing or 2π for the stance). Similarly, when the transition occurs, the phase is reset. Consequently, the leg controller is actually sensor-dependent.

Appendix C

Trajectory generation

C.1 Swing phase trajectory

The trajectory $\boldsymbol{r}_{sw}(\phi)$ must satisfy the following boundary conditions:

$$\begin{array}{rclrcl} \boldsymbol{r}_{sw}(0) & = & \boldsymbol{r}_{LO} & \boldsymbol{r}_{sw}(\hat{\phi}_{AEP}) & = & \hat{\boldsymbol{r}}_{AEP} \\ \boldsymbol{v}_{sw}(0) & = & \boldsymbol{v}_{LO} & \boldsymbol{v}_{sw}(\hat{\phi}_{AEP}) & = & \hat{\boldsymbol{v}}_{AEP} \end{array} \tag{C.1}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are respectively the position and speed (derivative with respect to time) vectors. Reference speed at AEP $\hat{\boldsymbol{v}}_{AEP}$ is set to $\mathbf{0}$ in order to allow for a smooth landing. The speed profile has to be continuous at LO to prevent a sudden undesirable generation of torque that could destabilize the model. This done by taking as an initial condition the desired value of the speed, as well as the position, of the foot when the transition from stance to swing is triggered. Finally, the trajectory must insure a sufficient toes clearance (h_{tc}).

A cycloid trajectory is used to generate the basic stepping-forward motion and is superposed with two other functions in order to satisfy the boundary conditions:

- the x component of a cycloid trajectory to account for the height difference between the LO and the AEP positions $\hat{r}_{AEP,y} - r_{LO,y}$
- a decreasing linear function to account for the initial speeds. The rate of vanishing of the influence of the initial speeds is set using the parameters a_1.

If t_{LO} is the time at LO, t the current time, ϕ the current phase, ω the angular frequency and using the following definitions:

$$\tau = \phi/\hat{\phi}_{AEP} \tag{C.2}$$
$$\dot{\tau} = \omega/\hat{\phi}_{AEP} \tag{C.3}$$
$$\Delta x = \hat{r}_{AEP,x} - r_{LO,x} \tag{C.4}$$
$$\Delta y = \hat{r}_{AEP,y} - r_{LO,y} \tag{C.5}$$
$$\phi_1 = a_1 \cdot \hat{\phi}_{AEP} \tag{C.6}$$
$$c_x(\xi) = (\xi - \frac{1}{2\pi} sin(2\pi\xi)) \tag{C.7}$$
$$c_y(\xi) = (1 - cos(2\pi\xi)) \tag{C.8}$$

the x components of the position and speed of $\boldsymbol{r}_{sw}$ are computed the following way:

$$\dot{p}_x = max(\frac{\phi_1 - \phi}{\phi_1} \cdot v_{LO,x}, 0) \tag{C.9}$$
$$p_x = \int_{t_{LO}}^{t} \dot{p}_x(\xi)\, \mathrm{d}\xi \tag{C.10}$$
$$q_x = \Delta x - p \tag{C.11}$$
$$r_{sw,x} = r_{LO,x} + q_x \cdot c_x(\tau) + p_x \tag{C.12}$$
$$v_{sw,x} = q_x \cdot \dot{\tau} \cdot c_y(\tau) + \dot{p}_x(1 - c_x(\tau)) \tag{C.13}$$

and the y components as:

$$\dot{p}_y = max(\frac{\phi_1 - \phi}{\phi_1} \cdot v_{LO,y}, 0) \tag{C.14}$$
$$p_y = \int_{t_{LO}}^{t} \dot{p}_y(\xi)\, \mathrm{d}\xi \tag{C.15}$$
$$q_y = \Delta y - p_y \tag{C.16}$$
$$r_{sw,y} = r_{LO,y} - \frac{h_{tc}}{2} \cdot c_y(\tau) + q_y \cdot c_x(\tau) + p_y \tag{C.17}$$
$$v_{sw,y} = -\pi \cdot h_{tc} \cdot \dot{\tau} \cdot sin(2\pi\tau) + q_y \cdot \dot{\tau} \cdot c_y(\tau) + \dot{p}_y(1 - c_x(\tau)) \tag{C.18}$$

with $a_1 = 0.25$ and h_{tc}= 2 cm.

C.2 Stance phase trajectory

Using the definitions:

$$\tilde{\phi} = \phi - \hat{\phi}_{AEP} \tag{C.19}$$
$$\tilde{\phi}_{st} = 2\pi - \hat{\phi}_{AEP} \tag{C.20}$$
$$\tag{C.21}$$

the trajectory $\boldsymbol{r}_{st}(\tilde{\phi})$ must satisfy the following boundary conditions:

$$\begin{array}{rclcrcl} \boldsymbol{r}_{st}(0) & = & \boldsymbol{r}_{TD} & & \boldsymbol{r}_{st}(\tilde{\phi}_{st}) & = & \hat{\boldsymbol{r}}_{PEP} \\ \boldsymbol{v}_{st}(0) & = & \boldsymbol{v}_{TD} & & \boldsymbol{v}_{st}(\tilde{\phi}_{st}) & = & \hat{\boldsymbol{v}}_{PEP} \end{array} \tag{C.22}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are respectively the position and speed (derivative with respect to time) vectors. $\boldsymbol{r}_{TD}$ is measured when touchdown occurs and the initial speed $\boldsymbol{v}_{TD}$ is set to $\mathbf{0}$. As mentioned in Section 4.4.2.2, $\hat{\boldsymbol{r}}_{PEP}$ and $\hat{\boldsymbol{v}}_{PEP}$ are given by:

$$\begin{array}{rclcrcl} \hat{r}_{PEP,x} & = & r_{TD,x} - \hat{L}_{str} & & \hat{v}_{PEP,x} & = & -\frac{\hat{L}_{str}(1-\hat{\beta})}{\hat{\beta}\hat{T}_{sw}} \\ \hat{r}_{PEP,y} & = & \hat{H} & & \hat{v}_{PEP,y} & = & 0 \end{array} \tag{C.23}$$

The x and y components of $\boldsymbol{\nu}$, the derivative of $\boldsymbol{r}_{st}$ with respect to the phase, are represented in Figure C.1, using the following definitions:

$$\tilde{\phi}_{td} = a_{td} \cdot \tilde{\phi}_{st} \tag{C.24}$$

$$\tilde{\phi}_{m} = a_{m} \cdot \tilde{\phi}_{n} \tag{C.25}$$

$$\tilde{\phi}_{d} = a_{d} \cdot \tilde{\phi}_{n} \tag{C.26}$$

$$\tag{C.27}$$

where $\tilde{\phi}_n$ is such that $r_{st,x}(\tilde{\phi}_n) = 0$. If $b_x = \hat{L}_{str}/\tilde{\phi}_{st}$, $\tilde{\phi}_n$ is given by:

$$\tilde{\phi}_n = \begin{cases} \frac{2\ r_{TD,x}\ +\ b_x\ \tilde{\phi}_{td}}{2\ b_x} & \text{if } 2\ r_{TD,x} > b_x\ \tilde{\phi}_{td} \\ \sqrt{\frac{2\ \tilde{\phi}_{td}\ r_{TD,x}}{b_x}} & \text{if not} \end{cases} \tag{C.28}$$

Accordingly, if t_{TD} is the time at TD, t the current time, and ω the angular frequency, the x and y components of the trajectory are given by:

$$\nu_x(\tilde{\phi}) = \begin{cases} -\frac{\tilde{\phi}\ b_x}{\tilde{\phi}_{td}} & \text{if } 0 < \tilde{\phi} < \tilde{\phi}_{td} \\ -b_x & \text{if } \tilde{\phi}_{td} \leq \tilde{\phi} \end{cases} \tag{C.29}$$

$$r_{st,x}(t) = r_{TD,x} + \int_{t_{TD}}^{t} \nu_x(\tilde{\phi})\omega \,\mathrm{d}\xi \tag{C.30}$$

and:

$$\Delta y = \hat{r}_{PEP,y} - r_{TD,y} \tag{C.31}$$

$$b_y = \frac{\Delta y}{\tilde{\phi}_n(a_d(1-k) - \frac{a_m}{2}) + k\tilde{\phi}_{st}} \tag{C.32}$$

$$\nu_y(\tilde{\phi}) = \begin{cases} \frac{\tilde{\phi}\ b_y}{\tilde{\phi}_m} & \text{if } 0 < \tilde{\phi} < \tilde{\phi}_m \\ b_y & \text{if } \tilde{\phi}_m \leq \tilde{\phi} < \tilde{\phi}_d \\ b_y \cdot exp(log(\kappa) \cdot \frac{\tilde{\phi}-\tilde{\phi}_d}{\tilde{\phi}_{st}-\tilde{\phi}_d}) & \text{if } \tilde{\phi}_d \leq \tilde{\phi} < \tilde{\phi}_{st} \\ 0 & \text{if } \tilde{\phi}_{st} \leq \tilde{\phi} \end{cases} \tag{C.33}$$

$$r_{st,y}(t) = r_{TD,y} + \int_{t_{TD}}^{t} \nu_y(\tilde{\phi})\omega \, \mathrm{d}\xi \tag{C.34}$$

with $\{a_{td}, a_m, a_d, \kappa, k\} = \{0.1, 0.2, 0.7, 0.01, 0.215\}$.

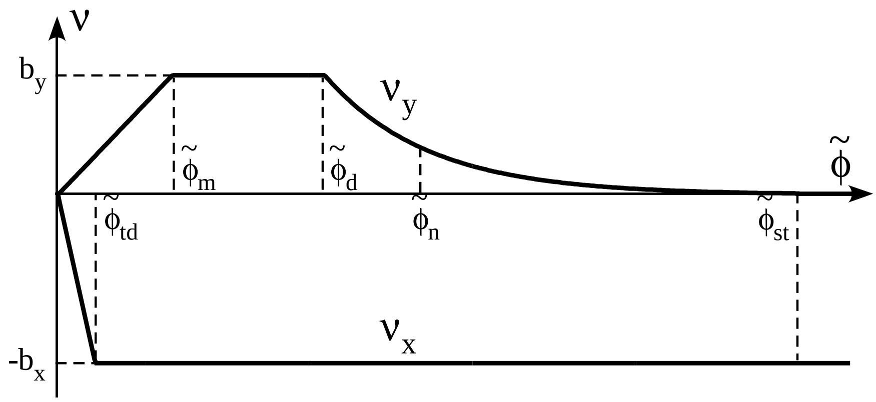


FIGURE C.1: ν_x and ν_y functions.

C.3 Inverse kinematic model

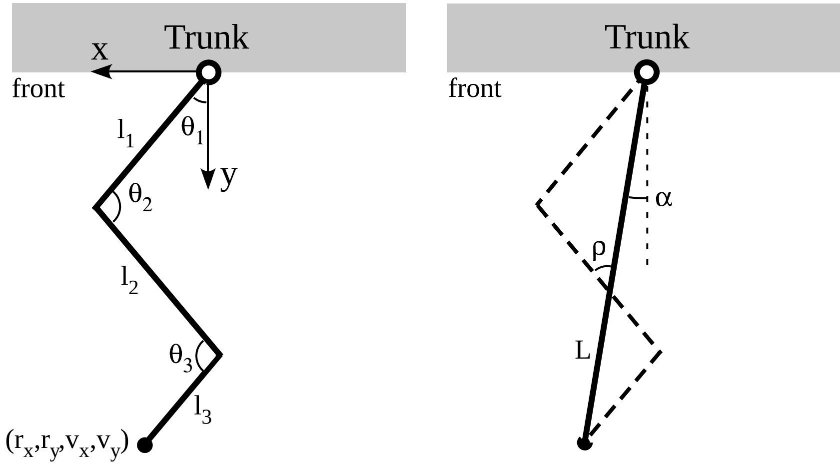


FIGURE C.2: Parameters of the inverse kinematics model

If α, L and ρ are defined as in Figure C.2, the joints angles (θ_j) and angular speeds ($\dot{\theta}_j$) are computed the following way (as the leg is tri-segmented, the constraint that the knee and ankle joints angles are identical, i.e. $\theta_2 = \theta_3$, is used):

$$\alpha = atan(r_x/r_y) \tag{C.35}$$

$$L = \sqrt{r_x^2 + r_y^2} \tag{C.36}$$

$$\theta_2 = \theta_3 = acos\left(\frac{(l_1^2 + l_2^2 + l_3^2 + 2l_1 l_3) - L^2}{2l_2(l_1 + l_3)}\right) \tag{C.37}$$

$$\rho = atan\left(\frac{(l_1 + l_3)sin\theta_2}{l_2 - (l_1 + l_3)cos\theta_2}\right) \tag{C.38}$$

$$\theta_1 = \pi + \alpha - \theta_2 - \rho \tag{C.39}$$

$$\dot{\alpha} = \frac{v_x r_y - v_y r_x}{r_x^2 + r_y^2} \tag{C.40}$$

$$\dot{L} = \frac{v_x r_x - v_y r_y}{\sqrt{r_x^2 + r_y^2}} \tag{C.41}$$

$$\dot{\theta}_2 = \dot{\theta}_3 = \frac{L\dot{L}}{l_2(l_1 + l_3)sin\theta_2} \tag{C.42}$$

$$\dot{\rho} = \frac{(l_1 + l_3)(l_2 cos\theta_2 - (l_1 + l_3))\dot{\theta}_2}{l_2^2 + (l_1 + l_3)^2 - 2l_2(l_1 + l_3)cos\theta_2} \tag{C.43}$$

$$\dot{\theta}_1 = \dot{\alpha} - \dot{\theta}_2 - \dot{\rho} \tag{C.44}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are respectively the position and speed of the foot (provided by the trajectory generation).