Adaptive Running of a Quadruped Robot Using Delayed Feedback Control

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Abstract— We report on the design and stability analysis of a simple quadruped running controller that can autonomously generate steady running with good energy efficiency and suppress such disturbances as irregularities of terrain. The self-stabilization property of the mechanical system is the inspirational source for our idea. We propose an original Delayed Feedback Control (DFC) approach based on measurements of the stance phase period obtained from contact sensors. Here, the DFC approach will not only stabilize running locomotion around the fixed point, but also result in transition from the stand state to the steady bounding running. Finally, we show several simulation results on different terrain (e.g., flat and step) to characterize the performance of the proposed controller. MPEG footage of simulations can be seen at: http://www.kimura.is.uec.ac.jp/running

I. INTRODUCTION

So far many studies [1], [2] have been conducted on the running locomotion of legged robots, of which the Raibert's monopod, biped and quadruped robots are the most well-known [3]. While Raibert's work has laid the foundation for studies in running locomotion, there remain the tasks of developing self-contained legged running robots, realizing running locomotion with even better efficiency and autonomous adaptation on irregular terrain.

Stabilizing a regular running locomotion can be considered that each state variable of the running locomotion converges at a fixed point. By analyzing the stability of the fixed point on *Poincare* map, many studies [4]–[9] have proved that the self-stabilization property of robots' mechanical systems could help robots to run at a relatively high-level of stabilities. In these model-studies and dynamics simulations, Blickhan et al. took advantage of the model called as Spring-Loaded Inverted Pendulum(SLIP) to explain that running on flat terrain can continue without any controls if the touchdown angle always satisfies a desired value [5]. However, self-stabilization didn't give robots the capability of autonomously adapting to changes in operation states (e.g., shift from the stand state to steady running state) or terrains (e.g. slope and step). Then the stabilization approach based on the information of sensors becomes very important. Recently, Orin et al. realized high-speed dynamic running gaits on a quadruped robot using an evolutionary search algorithm in dynamic simulations [10]. In our study, we want to design a simple quadruped running controller that is capable of autonomously generating the steady running locomotion with good energy efficiency and

suppressing such disturbances as irregularities of terrain.

In this paper, we will focus our discussion on three things. First, we consider qusi-passive running with the bounding gait in a sagittal plane compliant model and numerically seek fixed points of this running locomotion. Next, we treat friction and collision between legs and the ground as a disturbance surrounding the fixed point and suppress this disturbance by proposing our original DFC approach. Finally, we show several simulation results on different terrains to characterize the performance of the simple quadruped running controller.
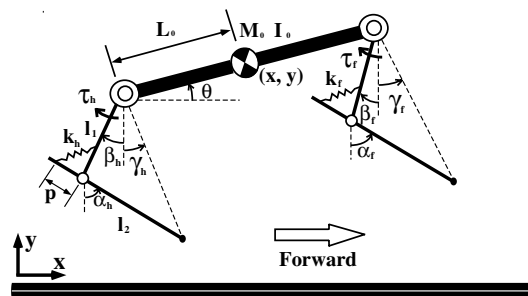
II. DYNAMIC MODEL AND BOUNDING GAIT

Fig. 1.   The sagittal plane model of a quadruped robot.

We constructed a sagittal plane compliant model of a quadruped robot to analyze bounding running. As shown in Fig. 1, the dynamic model is composed of a rigid torso and a pair of spring-loaded two-segment legs.In this paper, the height of the center of gravity when the robot stands on the initial ground (reference plane) is defined as the origin of potential energy. Then the whole (potential) energy of the system is called as "energy relative to reference plane".We study a bounding gait that contains four phases in one cycle of running locomotion in this paper, as illustrated in Fig. 2.

III. STABILIZATION OF STEADY RUNNING

A. Quasi-passive Running without Friction and Collision

The steady quasi-passive running locomotion can be formulated by a return map called as *Poincare* map and achieved by realizing the stabilization in a discrete dynamical system expressed by the *Poincare* map. Namely, Seeking the solutions of the *Poincare* map (i.e., fixed points) is significantly important. In our study, we first choose the state variables at the apex height in the extended
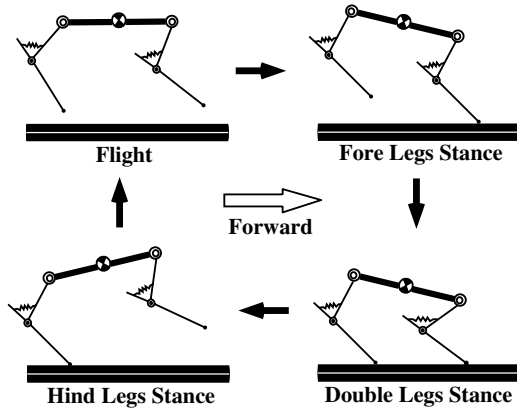
Fig. 2. Phase transitions of the sagittal plane bounding gait.

flight phase $p = [y_p, \theta_p, \dot{x}_p, \dot{\theta}_p]^T$ as a reference point of the $Poincare$ map and set the touchdown angles of fore and hind legs $q = [\gamma_f^{td}, \gamma_h^{td}]^T$ as the control input of the system. Second, we pay attention to the reference point $p[n]$ and control input $q[n]$ in the nth step in the discrete dynamical system defined in (1).

$$p[n+1] = \mathcal{P}(p[n], q[n]) \tag{1}$$

The equilibrium status p^*, q^* satisfies the next formula.

$$p^* = \mathcal{P}(p^*, q^*) \tag{2}$$

Here, if all the eigen-values of the $Jacobian$ matrix relating to the equilibrium status p^*, q^*

$$J = \frac{\partial \mathcal{P}(p, q)}{\partial p} \tag{3}$$

have magnitude less than one, p^* is an asymptotically stable fixed point. Finally, we use a well-known $Newton-$ $Raphson$ algorithm to numerically seek the combination of the fixed point p^* and the control input q^* from several initial values [8].

Although the quasi-passive running locomotion has energy conservation, it is possible that the running locomotion becomes unstable if there exists the transition between the potential energy and the kinetic energy at the apex height. Fortunately, since the running locomotion surrounding the fixed point p^* is asymptotically stable, i.e., the system has the self-stabilization property, stable running with the bounding gait can be realized simply by driving the touchdown angle to a desired value q^* without another complex control (e.g., adjusting the touchdown angle by measuring the forward speed and jump-height [3]). So, it is possible that the control system become extremely simple.

As a matter of fact, there exist a lot of fixed points and touchdown angles corresponding with these fixed points [6], [8]. In this paper, we adopt $p^* = [0.2, 0, 0.95, -1.52]^T$ as a typical fixed point and the touchdown angles $q^* = [0.524, 0.838]^T$ corresponding with this fixed point. Another feature quantities are expressed in Table I.

B. Application of Energy Reference Control with Friction and Collision

Although theoretically, the above-mentioned study about quasi-passive running on flat terrain can effectively demon-
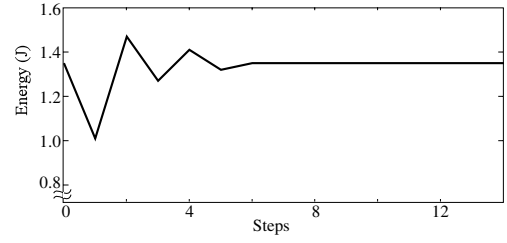
(a) Energy of the bounding quadruped at the apex
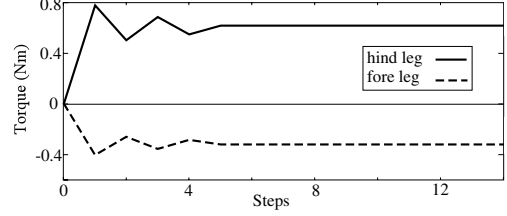
(b) Hip joint torque in the stance phase

Fig. 3. Results of the energy referenced control.

strate the self-stabilization property of the mechanical system, it appears inadequate when it comes to practical application since quasi-passive running is only an ideal locomotion. In some practical application of robots, we have to confront several difficulties posed by real environments.

Asano et al. decided a desired energy state based on a biped model of Passive Dynamic Walking(PDW) and proposed a stabilization approach called Energy Reference Control(ERC) for biped walking on flat terrain to adjust the torque of each leg based on the desired energy state [12]. In our study, we also used the ERC approach where the energy state at the apex height follows a desired energy state E^* calculated from a fixed point relating to quasi-passive running locomotion.However, in the calculation of the energy state, we only considered the model in which the mass of the robot is converged on the torso and the kinetic energy of swing legs is negligible relative to the kinetic energy of the torso in order to simply calculate the energy state of the system and effectively compare with the energy state of quasi-passive running.

We adopted the values of the fixed point listed in Table I and realized the ERC approach in a simulation. Fig. 3 shows the energy state at the apex height and the torques applied in hip joints in the stance phase. Here, whenever friction and collision cause the energy loss, energy is inputted by the ERC approach. As shown in Fig. 3, the system can recover the initial energy state and become the steady state in the sixth step. As a result, the quadruped robot is able to run with state variables such as those of the fixed point. Consequently, we conclude that the torques of hip joints during the sthace phase period in the steady state, $\tau_f^{st} = -0.32(Nm)$ and $\tau_h^{st} = 0.62(Nm)$, are essential for the compensation of the energy loss.

TABLE I

THE CHARACTERISTICS OF THE FIXED POINT

y_p (m)	0.2	θ_p (rad)	0
$\dot{x}_p$ (m/s)	0.95	$\dot{\theta}_p$(rad/s)	-1.52
γ_f^{td} (rad)	0.524	γ_h^{td} (rad)	0.838
energy (J)	1.36	cyclic period (s)	0.292
eigen values of J	0.0003, 0.0005, -0.032, 0.0213		

C. Application of Delayed Feedback Control with Friction and Collision

Delayed Feedback Control(DFC) can be considered that: in the next discrete dynamical system,

$$\boldsymbol{x}[n+1] = \mathcal{F}(\boldsymbol{x}[n], \boldsymbol{u}[n]) \tag{4}$$

$$\boldsymbol{y}[n] = \mathcal{G}(\boldsymbol{x}[n]) \tag{5}$$

the unstable fixed point $\boldsymbol{x}^*$ is stabilized by the control strategy such as:

$$\boldsymbol{u}[n] = \mathcal{K}(\boldsymbol{y}[n] - \boldsymbol{y}[n-1]) \tag{6}$$

Since the feedback described as divided difference is adequate, as expressed by (6), we believe that the stabilization of running can be realized without any deired trajectories (e.g., the desired energy state).

IV. CONSIDERATIONS FOR THE REALIZATION OF QUADRUPED BOUNDING RUNNING

A. Problems of Previous Approaches

We discover some problems with respect to these previous approaches described in **III-B** and **III-C**.

   (a) Under the ERC approach,the desired energy state must be previously determined. Due to the model error and so on, the desired energy state calculated based on the simulation or analysis is apparently improper if used in practical application.

   (b) When we measure the jump-height and forward speed by an acceleration sensor in running of a real robot, we have to confront several troubles (e.g., integration error, noise and drift). In real experiments, we are indeed unable to accurately calculate the energy of the system. Therefore, it is extremely difficult to use the feedback based on the energy state.

   (c) It becomes more important for practical application to construct a mechanism that enables the motion to shift from the stand state to the steady state.

In order to solve these above-mentioned problems, for one thing, we propose our original DFC approach based on the stance phase period change. For another, we design a rhythm generator, which is capable of adjusting the period and phase of running, for generating the bounding gait in the control system.

B. Adjustment of Energy Considering Sensing Problems

Cham et al. adjusted the stride period by measuring ground contact information and then realized the high-speed running locomotion of a hexapod robot over irregular terrain [7]. In their research, they found that it was much easier to measure the stride period through a binary switch atteached to the robot's feet than to measure the slope of the robot's torso by an angular velocity sensor. Thus adjusting the energy input period by a contact sensor is easier to apply to a real robot than that adjusting torque based on the slope of the torso. Although the DFC approach formulated

in (6) is able to stabilize the energy of the system, we still select the stance phase period, which can be accurately measured, as $\boldsymbol{y}$ similar to Cham's viewpoints since it is difficult to accurately calculate the energy of the running locomotion, as described in **IV-A**.

C. Generation of the Gait and Energy Input for Bounding Running

We know that the issure of bounding running about the transition from the stand state to the steady state is mostly classified into the generation of the gait and the energy input. And there exist some siginificant relations between the two issues.

First, as described in **III-B** and **III-C**, there is no a rhythm generator capable of explicitly generating the rhythm of motion in the above-mentioned two stabilization approaches (i.e., ERC and DFC), which could enable state variables to converge at the steady state from the initial non-static condition. Second, Buehler et al., determining constant torques to drive legs to a sweep limit angle during the stance phase period and adopting a PD controller to drive legs to a touchdown angle during the swing phase period, realized the transition from the stand state to the steady state with several running gaits on their quadrued robot, Scout II, without a rhythm generate [11].

Through our study, we believe that the use of a rhythm generator in the control system of a quadruped robot enjoys some advantages. For one thing, our energy efficient stabilization approach will enable state variables to converge at an asymptotically stable fixed point of quasi-passive running. For another, this rhythm generator, combined with our method in adjusting torques of hip joints in the stance phase (as shown in the next section), will improve the robot's anti-disturbance capability. But, since the rhythm of motion is mostly generated by spring mechanisms in the steady state, we conclude that the rhythm generator plays an unimportant role in the steady state.

We take advantage of a rhythm generator and a torque controller to realize the generation of the gait and the energy input in this paper. As a result, the motion is converged at an asymptotically stable fixed point of quasi-passive running.

V. TRANSITION FROM THE STAND STATE TO THE STEADY RUNNING

In fact, the most desirable control system is able to transfer state variables from the stand state to an unknown fixed point that satisfies a given apex height and forward speed. However, since the fixed points are different according to the different touchdown angles $\boldsymbol{q} = [\gamma_f^{td}, \gamma_h^{td}]^T$, we previously set a fixed point $\boldsymbol{p}^*$, which satisfies a given apex height and forward speed, as the desired state and used the touchdown angle $\boldsymbol{q}^*$ corresponding with the fixed point as information of the fixed point. Thus, state variables (e.g., period of running, leg phase difference, energy state,
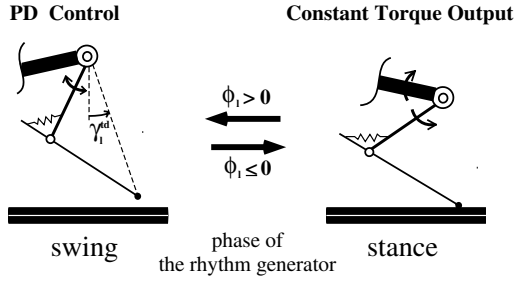
Fig. 4. Switching of the hip joint controller according to the output phase: ϕ_l of the rhythm generator.

apex height, forward speed and pitch angular velocity) that express the bounding gait will autonomously shift from the stand state to the steady state.

A. Generation of Leg Phases and Torque Output

1) Rhythm Generator: In the rhythm generator, we define the phase of each leg in the nth step ϕ_l, as described in (7). Here, the robot takes advantage of the leg phase to switch the torque controller. The timing for each leg to switch between the stance and swing phase is: $\phi_l > 0$:swing phase, $\phi_l \leq 0$:stance phase. (Fig. 4)

$$\phi_l = \sin(\omega_l[n]t + \psi_l) + \phi_{0l}, \quad \omega_l[n] = \frac{2\pi}{T_l[n]} \quad (7)$$

Here, $T_l[n]$ and $\omega_l[n]$ are the period and the angular frequency of leg l in the nth step, respectively. The initial phase ψ_l is defined for generating the gait. The offset ϕ_{0l} decides the duty factor. $T_l[n]$ is calculated by using the DFC approach described in **V-B**.

2) Torque Controller: Depending on the leg phase ϕ_l generated by the rhythm generator, different control actions are assigned by each high-level controller as shown in Fig. 4. More specifically, A PD controller described in (8) is performed during the swing phase period ($\phi_l > 0$).

$$\tau_l(t) = -K_p(\gamma_l - \gamma_l^{td}) - K_d \dot{\gamma}_l \quad (8)$$

During the stance phase period ($\phi_l \leq 0$), a constant torque $\tau_l^{st}[n]$ of the hip joint in each leg is output, as expressed by (9).

$$\tau_l(t) = \tau_l^{st}[n] \quad (9)$$

In the control action of the swing phase, γ_l^{td} is the touchdown angle that comes from the information of the fixed point (Table I). K_p and K_d are the gains of a PD control. In the control action of the stance phase, a torque DFC approach described in **V-B** decides the constant torque of the hip joint in each leg.

TABLE II

THE PARAMETER VALUES OF THE CONTROLLER USED IN

SIMULATIONS.

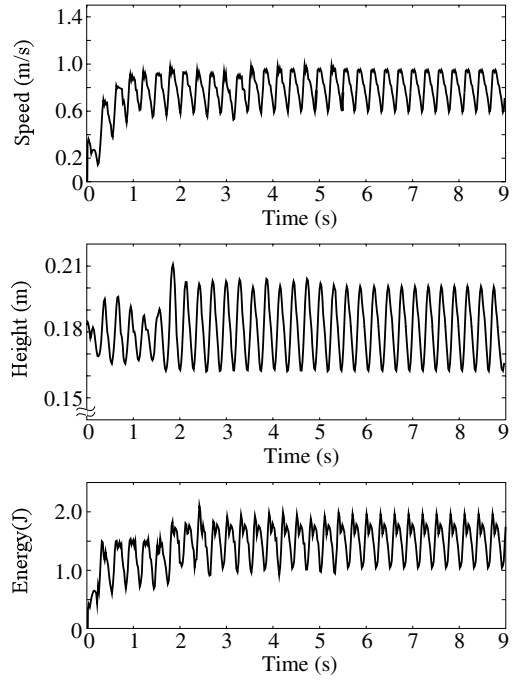| parameters	value	parameters	value
ψ_f	0	ψ_h	π
ϕ_{0f}	0.16	ϕ_{0h}	0.09
γ_f^{td}(rad)	0.524	γ_h^{td}(rad)	0.838
$K_{DF\cdot T}$	0.12	$K_{DF\cdot\tau}$	6.8
K_p(N·m/rad)	1.2	K_d(N·ms/rad)	0.02

Fig. 5. DFC in the transition from standing to steady bounding.

B. Converge at a Fixed Point based on DFC

We take advantage of next two equations to express the $\boldsymbol{x}$ and $\boldsymbol{y}$ in the discrete dynamical system described in (4) and (5).

$$\boldsymbol{x}[n] = [T_f[n], T_h[n], \tau_f[n], \tau_h[n]]^T \quad (10)$$
$$\boldsymbol{y}[n] = [t_f^{st}[n], t_h^{st}[n]]^T \quad (11)$$

Here, $t_l^{st}[n]$ represents the nth stance phase period measured by a binary contact sensor. As shown in **IV-B**, we use this stance phase period and propose the next DFC approaches.

$$T_l[n+1] = T_l[n] - K_{DF\cdot T}(t_l^{st}[n] - t_l^{st}[n-1]) \quad (12)$$
$$\tau_l^{st}[n+1] = \tau_l^{st}[n] - \delta(l)K_{DF\cdot\tau}(t_l^{st}[n] - t_l^{st}[n-1]) \quad (13)$$

$$\delta(l) = \begin{cases} -1, & l = f : foreleg \\ 1, & l = h : hindleg \end{cases}$$

Here, (12) and (13) respectively calculate the period of the leg phase and the torque of the hip joint during the stance phase period by using DFC. $K_{DF\cdot T}$ and $K_{DF\cdot\tau}$ are their DFC gains.

$K_{DF\cdot T}$ and $K_{DF\cdot\tau}$ in the proposed DFC approaches are decided by trial and error in simulations described in **V-C**. For one thing, the map $\mathcal{G}$ in (5), where $\boldsymbol{y}[n]$ is calculated by $\boldsymbol{x}[n]$, is extremely complex. For another, it is very difficult to analytically seek the DFC gains capable of enabling state variables to converge at a fixed point.

C. Simulation Results

In this section, we present some simulation results of bounding running on flat terrain. In order to generate the bounding gait, we should properly decide the initial values of the DFC approach. Here, we adopt $\{T_f[0], T_h[0], \tau_f[0], \tau_h[0]\} = \{0.20, 0.69, -1.0, 1.3\}$ as the
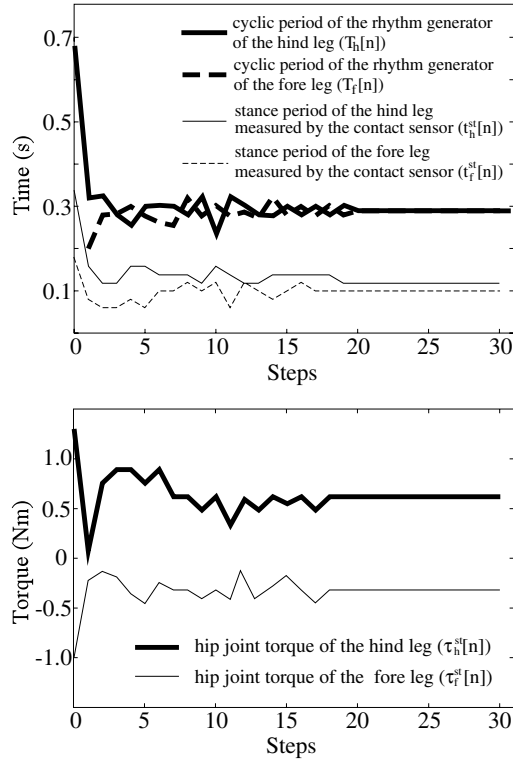
Fig. 6. DFC in the transition from standing to steady bounding.

Fig. 7. Bounding up a step 2 (cm) in height without DFC.

initial condition of the DFC approach expressed by (12) and (13). As described in **V-A**, the quadruped robot, according to ψ_l, begins to run with the initial condition where fore legs are in the swing phase and hind legs are in the stance phase. In this case, the initial values of $T_h[0]$ and $\tau_h[0]$ are much larger than those in the steady state in order to provide the sufficient kinetic energy during the first stance phase period of hind legs.

Fig. 5 shows the forward speed, jump-height and energy of the system as function of time. As shown in this figure, the apex height and forward speed in the steady state are similar to the state of the fixed point of quasi-passive running. If we also use the same calculation method as described in **III-B** where the mass of the robot is converged on the torso and the kinetic energy of swing legs is negligible relative to that of the torso, we discover that the energy states of the system in both cases are very similar.

Fig. 6 illustrates $t_l^{st}[n]$, $T_l[n]$, $\tau_l^{st}[n]$ relating to the DFC approaches expressed by (12) and (13). As shown in this figure, the DFC approach based on the stance phase period change is capable of stabilizing the period of the rhythm generator and the torque of the hip joint during the stance phase period. Such torque outputs concur with the results (in Fig. 3) achieved from the ERC approach described in **III-B**. So, the torque DFC approach expressed by (13) enables torques to converge on necessary minimum values that only compensate the energy loss caused by friction and collision.

VI. ANTI-DISTURBANCE CAPABILITY

First, we demonstrate that self-stabilization properties can also effectively work even when friction and collision
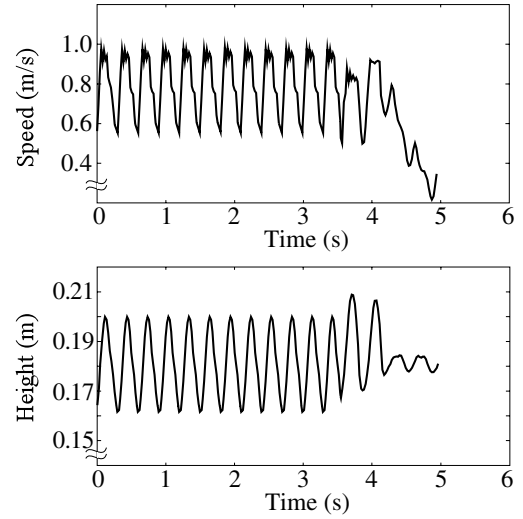
cause energy losses. The robot begins to run from initial conditions that satisfy state variables of the fixed point (Table I), without a rhythm generator(i.e., no feedback). In about 2(sec), there exists a small and temporary disturbance 0.034(rad) on the touchdown angle during the swing phase period. Although friction and collision cause energy losses of the system in this simulation, the self-stabilization property of the system is capable of suppressing the disturbance. Consequently, the robot can maintain the constant forward speed and jump-height in running by driving the touchdown angle to a desired value, without directly measuring the forward speed and jump-height for controlling them.

Next, by using the same control method as in the previous simulation, we realize the simulation where the robot runs on the initial ground (reference plane) with the same initial condition at first and runs on a 2(cm) step ground (touchdown plane) in 3.5(sec).As shown in Fig. 7, the bounding running becomes unstable since there is no DFC for a gait disorder and decreases of the "energy relative to touchdown plane". Therefore, the self-stabilization property appears inadequate when it suppresses a disturbance where there exists energy change relative to the touchdown plane.

Finally, we take advantage of the proposed stabilization approach as described in **V**, and realize the simulation where the robot begins to run from the steady state and runs on a 2(cm) step ground in about 2(sec). As shown in Fig. 9, since $t_l^{st}[n]$ is changed when the robot runs over the step in the 7th step, DFC begins to work. More specifically, DFC outputs larger torques of hip joints in hind legs ($\boldsymbol{B}$) according to the longer stance phase period of hind legs ($\boldsymbol{A}$) in the 8th step, and provides the necessary energy input for jumping in about 2.3(sec) as shown in Fig. 8. At last, the gait, torque and so on converge at initial conditions in the 17th step through the transient states. In addition, Fig. 8 shows that the forward speed, jump-height and energy can also converge at initial conditions by adjustments based on DFC.

In this section, we prove that the proposed DFC approach is capable of generating the gait and providing the energy
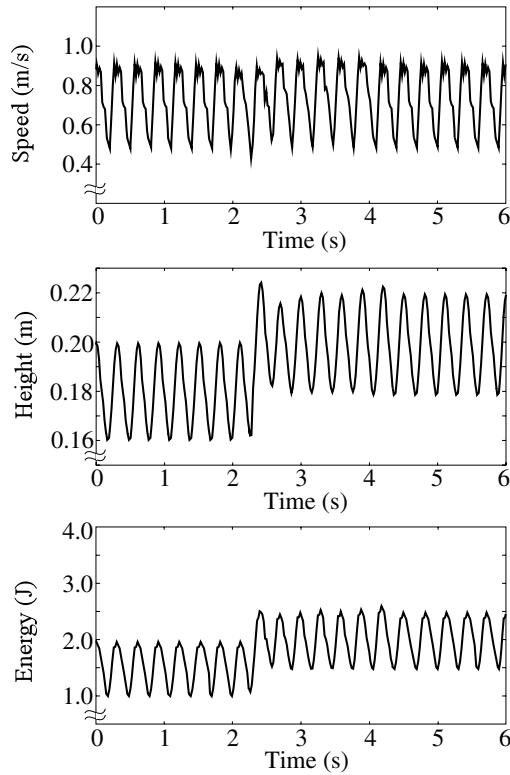
Fig. 8.   Bounding up a step 2 (cm) in height with DFC.

input. Thus, it is sufficient for autonomously adapting to the irregular terrain where the system has a temporary disturbance.

VII. CONCLUSION AND FUTURE WORK

In this paper, we only considered the touchdown angle of each leg as the information of the fixed point of quasi-passive running in a quadruped robot with spring mechanisms, and proposed our original DFC approach where we enabled the stance phase period change measured by a contact sensor to feedback to the rhythm generator and the torque controller. Especially, in order to apply the proposed stabilization approach to a real robot, we took into consideration some elements (i.e., the leg mass, viscous friction in joints and collision between legs and the ground) in our simulations, design a rhythm generator to generate the bounding gait, consider some stabilization methods without the desired energy state, and utilize the DFC approach based on the stance phase period change because it much easier and more accurate to measure the stance phase period by a contact sensor than to measure the forward speep or the slope of the torso by a gyro sensor.

As a result, the obtained running locomotion is similar to the motion surrounding the fixed point of quasi-passive running and has superior self-stabilization property and energy efficiency. Moreover, as demonstrated previously, the proposed simple quadruped running controller is also capable of adapting running over a setp where "energy relative to touchdown plane" temporarily increases or decreases. In our future work, we plan to realize the practical experiment of bounding running on a quadruped robot that is our platform of simulations.
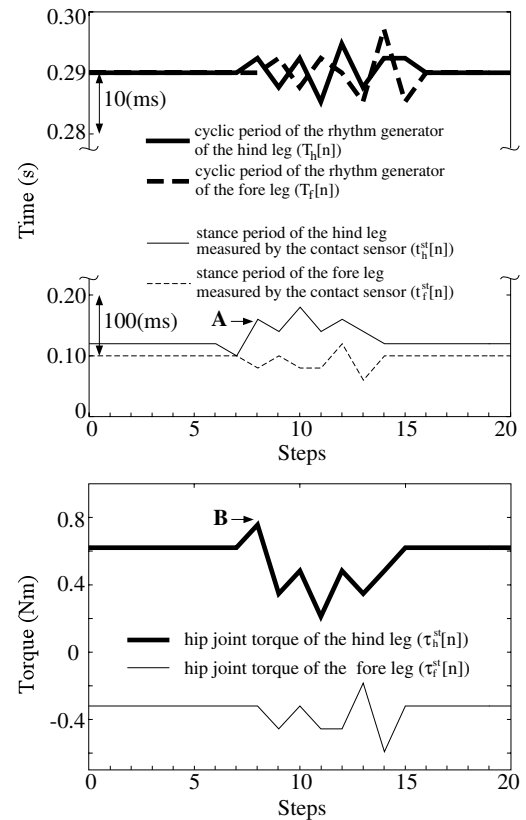

Fig. 9.   Bounding up a step 2 (cm) in height with DFC.

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