Stable Dynamic Walking of a Quadruped via Phase Modulations against Small Disturbances

Christophe Maufroy, Hiroshi Kimura and Kunikatsu Takase

Abstract—It is generally accepted that locomotion in animals is based on a trade-off between energy consumption and stability. However, this trade-off is the result of the interaction between complex mechanical and control systems. To gain insight into that issue, a step-by-step approach is needed.

In this study, as a first step to investigate three dimensional quadrupedal walking, we aim at establishing a control system as “minimal” as possible, able to realize stable dynamic walk. Using a simple mechanical structure, we realized dynamic walk with a distributed control system, made of four independent leg controllers whose swing and stance phases durations are modulated based on leg loading information. Phase modulations contribute to stabilize the posture in the frontal plane via automatic duty ratios adjustments that tend to compensate perturbations of the body rolling motion. By applying lateral perturbations, we found that the control system withstands well perturbations increasing the rolling motion amplitude, but is sensible to perturbations that suddenly decrease it, as the foreleg on the more loaded side is prevented to swing. Hence, we implemented an ascending coordination mechanism where the transition to swing in a hind leg promotes the same event in the foreleg. The duration of the subsequent foreleg swing phase is reduced to prevent excessive increase of the rolling motion amplitude. The resultant control system, although extremely simple, was able to realize dynamic walk resistant to small disturbances (lateral perturbations and terrain irregularities).

I. INTRODUCTION

Traditional methods for dynamic legged locomotion control are generally classified into Zero Moment Point (ZMP) based control and limit-cycle based control. ZMP based control is effective for controlling posture and low-speed walking of biped and quadruped [1][2], but is not good for medium or high-speed walking from the standpoint of energy consumption. In contrast, motion generated by limit-cycle based control [3], making use of the natural dynamics\(^1\) of the system, has superior energy efficiency, but there exists an upper bound of the period of the walking cycle, in which stable dynamic walking can be realized [4].

Based on this latter approach, quadrupedal dynamic walking on irregular terrain was realized with the robot Tekken [5] using a neural controller made of a Central Pattern Generator (CPG)[6] and a set of reflexes. However, it was not clear how much the phase modulations at the CPG level and the reflexes respectively contributed to the stability. Moreover, dynamic walk with a long cyclic period could not be realized because increasing the period caused large rolling motion of the body, leading to instabilities.

Stabilization of the rolling motion can be achieved by phase modulations which consist in modulations of the respective durations of the stance and swing phases of the legs during the walking cycle. Various implementations of phase modulations in simulation and on real robots have been reported. In [5], swing and stance phases durations are modulated using hip joint and body roll angles feedbacks. Foot contact information was used in [7][8] to modulate the swing phase duration using phase resetting at touchdown. On the other hand, it is known that in animals stance phase termination is regulated using leg loading information [9].

Using a 2D model of cat hind legs, [10] showed that this mechanism plays an essential role in the emergence of stable alternative stepping. In [11], leg loading was used to modulate swing and stance phases durations in a CPG control system. However, gait generation was considered at the CPG level (via the settings of the couplings between the leg oscillators) and the contribution of phase modulations to leg coordination and posture stabilization was not detailed.

In this contribution, we investigated in simulation how phase modulations based on leg loading information can stabilize rolling motion in order to realize stable dynamic walk with a control system as simple as possible. We found that a extremely simple control architecture, made of four independent leg controllers whose phase transitions are regulated using leg loading information and a single coordination mechanism linking ipsilateral legs, can generate dynamic walking patterns resistant to small disturbances (lateral perturbations and terrain irregularities).

II. MECHANICAL AND CONTROL SYSTEMS

A. Simulation model

This work has been partially supported by a Grant-in-Aid for Scientific Research on Priority Areas “Emergence of Adaptive Motor Function through Interaction among Body, Brain and Environment” from the Japanese Ministry of Education, Culture, Sports, Science and Technology.

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\(^1\)inverted pendulum motion, natural oscillation in compliant systems,...
summarizes the dimensions and masses of the model bodies. The distance between right and left hip joints is 12 cm. Controller (LC), whose structure is represented in Fig. 2. The control of this motion, referred to as leg loading, is primordial to insure postural stability in the frontal plane. In order to keep the model simple and to focus on the rolling motion stabilization mechanism provided by the phase modulations (see Sec. IV-A), the model does not have any joint around the roll axis. Hence the motion of each leg is two-dimensional and restricted to a plane parallel to the sagittal plane.

B. Leg Controller

Each leg is actuated by a control unit called the Leg Controller (LC), whose structure is represented in Fig. 2. Each LC has two phases: swing (sw) and stance (st). The transfer of activity between them is regulated using sensory information related to the load supported by the leg, or leg loading. Each LC is associated with a simple oscillator model, with a constant and unitary amplitude and a variable phase $\phi_i$, with the leg index $i$. The phase of the oscillator is used for the trajectory generation, as well as to express the phase relationship between the legs.

The position of the foot where the swing-to-stance transition is desired to happen is the anterior extreme position $\hat{r}_{AEP}$, while the position where it really happens is the liftoff position $r_{LO}$. Similarly, the position where the stance-to-swing transition is desired to happen is the posterior extreme position $\hat{r}_{PEP}$, while the position where it really happens is the touchdown position $r_{TD}$.

1) Phase dynamics: When a phase transition occurs, $\phi$ is first reset ((1),(3)), then increases constantly (until a maximum value for the swing phase) with a rate given by the angular velocity $\omega_i$ (2),(4)) . The parameters used for the phase dynamics are defined as follows:

$$\dot{\omega} = 2\pi(1-\tilde{\beta})/T_{sw} \quad \dot{\phi}_{AEP} = 2\pi(1-\tilde{\beta}) \quad \dot{\phi}_{PEP} = 0$$

where $\tilde{\beta}$ and $T_{sw}$ are respectively the nominal values of the duty ratio and the swing phase duration. $\omega_{mod}$ is used to modulate the swing phase duration (see Sec. V).

2) Foot trajectory generation: For each locomotion phase, the foot trajectory is computed between the initial measured position ($r_{LO}$ or $r_{TD}$) and the desired final position ($\hat{r}_{AEP}$ or $\hat{r}_{PEP}$), in a way that guaranties the continuity of the speed at the transitions. The swing and stance phases trajectories $r_{sw}(\phi)$ and $r_{st}(\phi)$ are encoded using the phase $\phi$ and the current desired foot position is updated as $\phi$ increases.

Taking into account that the angles of the LCs are used during the two-legged stance phases. The c ontrol of this motion, referred to as leg loading, is primordial to insure postural stability in the frontal plane. In order to keep the model simple and to focus on the rolling motion stabilization mechanism provided by the phase modulations (see Sec. IV-A), the model does not have any joint around the roll axis. Hence the motion of each leg is two-dimensional and restricted to a plane parallel to the sagittal plane.

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desired joint angles ($\hat{\theta}_j$) are computed using the inverse kinematics model of the leg (as the leg is tri-segmented, knee and ankle joints angles are set equal) and the joint torques $\tau_j$ are generated using a PD control law:

$$\tau_j = K_{P,j,k}(\hat{\theta}_j - \theta_j) + K_{D,j,k}(\dot{\hat{\theta}}_j - \dot{\theta}_j), \quad k \in \{sw, st\} \quad (9)$$

Regarding $r_{sw}(\phi)$, $\Delta$ is an adjustable offset of the AEP vertical coordinate, used to delay the stance phase of the forelegs compared to hind legs (see Sec. III-A.3). On the other hand, for $r_{st}(\phi)$, the $x$ component of the velocity $v_{st,x}$ is set constant during the stance phase (but for an initial constant acceleration period). For $v_{st,y}$, various profiles have been tested and lead to similar results as long as the velocity is large at the beginning of the stance phase and decreases fast enough afterward.

3) Transition conditions: The transitions between the swing and stance phases in each leg controller are regulated using conditions based on the normal ground reaction force $f_n$ measured by the force sensor, which is related to the leg loading. The force threshold $\chi$ was set to 10 N, which is slightly less than one quarter of the model weight. Additional conditions based on $\phi$ are added to prevent early transitions, just after the transfer from one phase to the other.

C. Simulation Environment

The simulation of the mechanical model and the control system was carried out using Webots, a commercial mobile robot simulation software developed by Cyberbotics Ltd (http://www.cyberbotics.com). For the ground reaction forces, the parameters were set so that the stiffness and damping of an equivalent spring-damper model would be equal respectively to $3.0e4 \text{ Nm}^{-1}$ and $2.0e3 \text{ Nsm}^{-1}$.

III. WALKING ON FLAT GROUND

A. Realization of the walk gait

1) Considerations about gaits: A gait is a locomotion pattern characterized by phase differences ($\in [0, 1]$) between the legs during their pitching motion. We will use $\gamma^{ctr}$ and $\gamma^{lat}$ to refer respectively to the phase differences between HL and HR on one hand, and between HL and FL on the other. The walk is a transversal gait between the trot and the pace, hence characterized by $\gamma^{ctr} = 0.5$ and $\gamma^{lat} \approx 0.25$.

2) Emergence of alternate coordination: The simplest control system configuration, i.e., four leg controllers operating independently, was first investigated. Even in that situation, coordinate locomotion patterns emerge and maintain in a broad range of parameters values when a rolling motion of a sufficient amplitude is induced at the beginning of the simulation. All the gaits observed are characterized by the alternating stepping of the right and the left legs, i.e., $\gamma^{ctr} = 0.5$ and $\gamma^{lat} \in [-0.25; 0.25]$.

The emergence of the alternate coordination can be explained the following way. As the swing phase of ipsilateral legs overlap, rolling motion is induced by the gravity, as the system is equivalent to an inverted pendulum during the two-legged support phase. As rolling motion is linked to the lateral motion of the body, after the touchdown of the swinging legs, the load due to the body weight is transferred laterally, in the same direction as the rolling motion (this load transfer mechanism will be referred to as lateral transfer of leg loading). This unloads the former supporting legs so that the transition to swing is triggered (when $f_n < \chi$). As a result, rolling motion is induced in the other direction and the same sequence of events repeat symmetrically. Hence, alternate coordination emerges as the result of the entrainment between the leg stepping, the rolling motion and the lateral transfer of leg loading.

3) Adjustment of $\gamma^{lat}$: If the same parameters are used for the fore and hind legs LCs, $\gamma^{lat}$ does not exceed 0.1, so that, to realize a walk gait, an additional mechanism is needed to delay the stepping of the forelegs relative to the hind legs. Although this could be done by introducing an explicit coupling between the LCs, we found that the adjustment of the relative values of $\Delta$ (see (5)) in the fore and hind legs LCs could also fulfill this task, with the benefit of keeping the LCs independent. As the body rotates around its roll axis due to the rolling motion, setting $\Delta^{v*} > \Delta^{H*}$ delays the touchdown of the forelegs compared to the hind legs, hence increasing $\gamma^{lat}$. Accordingly, walk gait could be realized with independent leg controllers (Fig. 3).

B. Modulation of the walking patterns

1) Cyclic period modulation: The cyclic period can be modulated by changing the nominal swing phase duration $T_{sw}$. As the period increases, the rolling motion amplitude becomes larger (being proportional to the square of the pe-
period [4]) and $\gamma_{lat}$ decreases, causing the gait to progressively change to a pace. To preserve the walk gait, we found that $\Delta F^*$ must be increased, while the stiffness of the legs during the stance must be decreased (by reducing $K_{Pj,st}$ and $K_{Dj,st}$ of the knee and ankle joints). Using this approach, walking patterns with cyclic period ranging from 0.30 s to 0.88 s ($T_{sw} \in [0.10, 0.35]$s and $\beta = 0.75$) could be realized.

2) Speed modulation: Speed modulation can be achieved by adjusting $\beta$, which adjusts the sweeping speed of the legs during the stance (see (6)). As the speed increases, both the period and the rolling motion amplitude were found to decrease. As a result, $\gamma_{lat}$ increased, causing the gait to get closer to a trot. Hence, $\Delta F^*$ needed to be decreased, while $K_{Pj,st}$ and $K_{Dj,st}$ of the knee and ankle joints had to be increased to maintain the same $\gamma_{lat}$ value. With these adjustments, the speed could be modulated from 0.08 to 0.72 m/s (0.33 to 3 body length/s).

IV. INDEPENDENT LEG CONTROLLERS: STABILITY EVALUATION

A. Stabilization provided by the phase modulations

The stabilization mechanism provided by the phase modulations based on leg loading information is schematically represented in Fig. 4, when considering the motion in the frontal plane. When the body rolling motion becomes asymmetric (i.e. $<\theta_{roll}> \neq 0$, with $< \ast >$ representing the average value over a period$^5$) due to a perturbation, the average load $< f_n >$ supported by the left and right legs differs. This results in the increase (resp. the decrease) of the stance phase duration as the leg is more (resp. less) loaded, due to the phase modulations. Hence the effective duty ratios $\bar{\beta}$ are automatically adjusted and the original difference between $< f_n >$ in the right and left legs is amplified to generate a stabilizing torque that tends to cancel the asymmetry.

B. Resistance ability against lateral perturbations

We subjected our model to lateral perturbations to assess the stability of the stepping patterns. A force, pushing the model to the left, was applied at the center of mass of the trunk during 0.1 s at four different timings during the walking cycle: at the onset of the swing phase of the left hind leg (HL), left foreleg (FL), right hind leg (HR) and right foreleg (FR). The experiments were carried out using two walking patterns with different cyclic periods:

- (SH) with $T_{tot} \approx 0.40$ s ($T_{sw} = 0.15$ s and $\beta = 0.75$)
- (LG) with $T_{tot} \approx 0.63$ s ($T_{sw} = 0.25$ s and $\beta = 0.75$)

Results are presented in Table II and shows that the ability to recover from perturbations when it occurs at the swing onset of either FL, HR or FR is relatively good (the maximum amplitude supported is typically higher than 10 N, except in the case of HR for (LG)). On the other hand, the model is relatively sensitive to perturbations applied at the onset the swing phase of HL.

$^5$Defined as: $f = \frac{\int_{ts}^{te} f(t) dt}{te-ts}$ where $ts$ and $te$ are respectively the start and end instants of the ongoing cycle, when taking a particular event as reference (for example the onset of the swing phase of the left hind leg).

For all timings, the perturbation is directed to the left of the model, which causes the angular velocity of rolling $\dot{\theta}_{roll}$ to decrease. Accordingly, the stance and swing phases durations adjustments due to the phase modulations are qualitatively similar (i.e. they result in $\beta^L < \beta^R$). The difference of performance is related to the influence of the perturbation on the amplitude of the body rolling motion and the associated lateral transfer of leg loading.

In the first case (i.e. FL, HR, FR), the perturbation causes $|\dot{\theta}_{roll}|$ to increase, leading to a temporary augmentation of rolling motion amplitude (see Fig. 5). As a result, the lateral transfer of leg loading is accelerated. The additional load supported by the left legs causes their stance phase to be extended, while the swing phase of the right legs is prolonged.

On the other hand, for HL, the perturbation causes $|\dot{\theta}_{roll}|$ to decrease, leading to a temporary diminution of the rolling motion amplitude (see Fig. 6). This tends to cancel the rolling motion and the associated lateral transfer of leg loading. Accordingly, the rate of unloading of the left foreleg decreases and, if the amplitude perturbation is large enough, $f_{FL}$ never becomes smaller than the threshold $\chi$, preventing the transition to the swing phase. This greatly disturbs the leg coordination, causing the model to fall forward as none of the forelegs is allowed to swing. This situation can be seen as a conflict between rhythmic motion control, which requires the leg to swing to preserve the coordination, and posture control in the frontal plane, which prevents a leg still supporting the body to swing for stability reasons.

<table>
<thead>
<tr>
<th>TABLE II</th>
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<tr>
<td>MAXIMUM AMPLITUDE OF THE LATERAL PERTURBATION SUPPORTED</td>
</tr>
<tr>
<td>(SH)</td>
</tr>
<tr>
<td>2 N</td>
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<tr>
<td>(LG)</td>
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V. ASCENDING COORDINATION MECHANISM

An additional mechanism is needed to solve the conflict and help the system to recover when the rolling motion amplitude is suddenly reduced. We implemented a two-fold ascending coordination mechanism that brings a solution to the main cause of failure (i.e. the foreleg cannot swing) while limiting the impact on the stability.

First, stance-to-swing transition in a hind leg (Hx) promotes the same event in the ipsilateral foreleg (Fx). The force threshold of the foreleg \( \chi^{Fx} \) is linearly increased as \( \phi^{Hx} \) increases during the swing phase of the hind leg, as follows:

\[
\chi^{Fx} = \begin{cases} 
\ddot{x} & \text{if } \phi^{Hx} < \phi_{acm} \\
\ddot{x} + \chi_{mod} & \text{if } \phi^{Hx} \in [\phi_{acm}; \hat{\phi}_{AEP}] \\
\dddot{x} & \text{if } \phi^{Hx} > \hat{\phi}_{AEP}
\end{cases}
\]

\[
\chi_{mod} = \frac{\phi^{Hx} - \phi_{acm}}{\hat{\phi}_{AEP} - \phi_{acm}} \cdot \chi_{ampl}
\]

where \( \phi_{acm} \) is the threshold over which the modulation occurs and \( \chi_{ampl} \) is the maximum value of the modulation. \( \phi_{acm} \) is set to be slightly bigger than the value of \( \phi^{Hx} \) at which the transition occurs in normal conditions. This part of the mechanism detects the disruption of the normal leg loading and unloading cycle (that prevents the foreleg to swing) and restarts the body rolling motion by forcing the transition to swing in the foreleg.

Second, the duration of the next swing phase of the foreleg is shortened, by setting \( \omega_{mod} \) of (2):

\[
\omega^{Fx}_{mod} = \lambda \cdot \frac{F_{n,LO}^{Fx} \cdot \ddot{x}}{\chi_{ampl}} 
\]

where \( F_{n,LO}^{Fx} \) is the normal ground reaction force measured by the force sensor of the foreleg at liftoff. If greater than \( \dddot{x} \), it indicates that the transition was triggered by the first part of the mechanism so that \( \theta_{roll} \) is closer to 0 than in the normal conditions (as the rolling motion amplitude is reduced). Hence, this part of the mechanism shortens the swing phase duration to avoid excessive increase of the roll angle in the direction of the swinging foreleg (the value of the roll angle at liftoff is estimated by the load supported by the foreleg at the transition).

While allowing the foreleg to swing, the two previous adjustments globally increase the duty ratio of the foreleg: the transition to the swing phase is still delayed compared to normal (hence the stance phase is extended) while the next swing phase duration is shortened. Moreover, the increase of the duty ratio is positively related with the increase of the load supported by the foreleg (i.e. with the intensity of the perturbation). Consequently, this mechanism works in cooperation with the stabilization provided by the phase modulations (its action is illustrated in Fig. 7 and the parameters settings given in Table III). Adding this mechanism leads to the improvement of the resistance to the perturbation at the onset of the swing phase in HL, up to 25 N and 16 N for (SH) and (LG) respectively.

Interestingly, the existence of a similar coordination mechanism, i.e. an ascending excitatory pathway linking the systems generating flexor bursts of ipsilateral fore and hind legs, was proposed in [12] to explain results obtained during walking on split-treadmill experiments with cats.

VI. WALKING ON UNEVEN TERRAIN

Finally, we evaluated the performances of the system when coping with two kinds of uneven terrain: elevated steps and slopes. For the elevated footsteps, three situations were considered: the model steps once on an elevated step with its right foreleg (foreleg case), with its right hind leg (hind leg case) or has to continuously walk with both its right legs on a lateral step (lateral step case). For the slopes, both up and down slopes were considered.

Results with and without the ascending coordination mechanism are presented in Table IV (respectively the max-

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**TABLE III**

<table>
<thead>
<tr>
<th>( \phi_{acm} )</th>
<th>( \chi )</th>
<th>( \chi_{ampl} )</th>
<th>( \lambda )</th>
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</thead>
<tbody>
<tr>
<td>0.55 ( \phi_{AEP} )</td>
<td>10 N</td>
<td>15 N</td>
<td>1</td>
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Phase modulations based on leg loading information contribute to stabilize the posture in the frontal plane through automatic duty ratios adjustments that tend to compensate perturbation-induced asymmetries in the body rolling motion. The efficiency of this stabilization mechanism was evaluated by applying lateral perturbations at various timings during the walking cycle. The control system was found to be globally resistant to perturbations increasing the rolling motion amplitude while being sensitive to perturbations decreasing it, as the transition to swing in the foreleg on the more loaded side is prevented in that case.

Consequently, we implemented an ascending coordination mechanism that promotes stance-to-swing transition in a foreleg by raising its force threshold after a certain time following the liftoff of the ipsilateral hind leg. When the transition is triggered by this mechanism, subsequent swing phase duration is reduced to prevent the excessive increase of the amplitude of the rolling motion. When adding this mechanism to the control system, the model achieved good resistance ability against lateral perturbations at all the timings considered and could cope with various terrain irregularities, both with a short and a longer walking periods.

In the future, we intend to test our control system on a real robot to validate the results obtained in simulations.

**REFERENCES**